Plasma Physics

1.1. What is Plasma?:

Any ionized gas cannot be called plasma, of course; there is always some small degree of ionization in any gas. A useful definition is as follows:

Plasma is a quasi neutral gas of charged and neutral particles which exhibits collective behaviour.

1.2 Plasma oscillations:

If the electrons in plasma are displaced from a uniform background of ions, electric fields will be built up in such a direction as to restore the neutrality of the plasma by pulling the electrons back to their original positions. Because of their inertia, the electrons will overshoot and oscillate around their equilibrium positions with a characteristic frequency known as the plasma frequency. This oscillation is so fast that the massive ions do not have time to respond to the oscillating field and may be considered as fixed. The open rectangles represent typical elements of the ion fluid, and the darkened rectangles the alternately displaced elements of the electron fluid. The resulting charge bunching causes a spatially periodic E field, which tends to restore the electrons to their neutral positions.

We shall derive an expression for the plasma frequency ω_p in the simplest case, making the following assumptions: (i) There is no magnetic field; (ii) there are no thermal motions (KT = 0); (iii) the ions are fixed in space in a uniform distribution; (iv) the plasma is infinite in extent; and (v) the electron motions occur only in the x direction.

We now consider the dynamic response of the plasma to n imposed charge separation. To study this, we displace electrons and ions and calculate the transient response. Since the electrons are much lighters the ions, we assume the ions are immobile.

Then, F = qE

$$m_e \frac{d^2x}{dt^2} = -e\frac{\rho x}{\varepsilon_0} \qquad (1)$$

$$m_e \frac{d^2x}{dt^2} = -\omega_p^2 x \qquad (2)$$

which is the equation of a simple harmonic oscillator with electron plasma frequency.

$$\omega_p^2 = \left(\frac{\rho e}{m_e \varepsilon_0}\right) = \left(\frac{n e^2}{m_e \varepsilon_0}\right) \qquad (3)$$

1.3 Debye's potential and Debye shielding:

A fundamental characteristic of the behavior of plasma is its ability to shield out electric potentials that are applied to it. Suppose we tried to put an electric field inside plasma by inserting two charged balls connected to a battery (Fig. 1.3). The balls would attract particles of the opposite charge, and almost immediately a cloud of ions would surround the negative ball and a cloud of electrons would surround the positive ball. (We assume that a layer of dielectric keeps the plasma from actually recombining on the surface, or that the battery is large enough to maintain the potential in spite of this.) If the plasma were cold and there were no thermal motions, there would be just as many charges in the cloud as in the ball, the shielding would be perfect, and no electric field would be present in the body of the plasma outside of the clouds. On the other hand, if the temperature is finite, those particles that are at the edge of the cloud, where the electric field is weak, have enough thermal energy to escape from the electrostatic potential well. The "edge" of the cloud then occurs at the radius where the potential energy is approximately equal to the thermal energy KT of the particles, and the shielding is not complete. Potentials of the order of KT/e can leak into the plasma and cause finite electric fields to exist there.

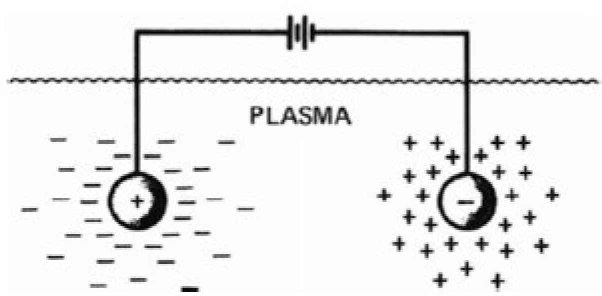

Fig. 1. Debye shielding

Let us compute the approximate thickness of such a charge cloud. Imagine that the potential ϕ on the plane x= 0 is held at a value ϕ_0 by a perfectly transparent grid (Fig. 1). We wish to compute $\phi(x)$. For simplicity, we assume that the ion–electron mass ratio M/m is infinite, so that the ions do not move but form a uniform background of positive charge. To be more precise, we can say that M/m is large enough that the inertia of the ions prevents them from moving significantly on the time scale of the experiment. Poisson's equation in one dimension is

$$\varepsilon_0 \nabla^2 \emptyset = \frac{d^2\emptyset}{dx^2} = -e(n_i - n_e) \qquad (z = 1) \qquad (4)$$

If the density far away is n_∞, we have, $n_i = n_\infty$

In the presence of a potential energy $q\phi$, the electron distribution function is

$$f(u) = A \exp[-\frac{\frac{1}{2}mu^2 + q\emptyset}{kT_e}] \qquad (5)$$

It would not be worthwhile to prove this here. What this equation says is intuitively obvious: There are fewer particles at places where the potential energy is large, since not all particles have enough energy to get there. Integrating f (u) over u, setting q = -e, and noting that $n_e(\phi \to 0) = n_\infty$; we find

$$n_e = n_\infty \exp\,(e\phi/kT_e) \qquad (6)$$

This equation will be derived with more physical insight. Substituting for n_i and n_e in Eq. (4), we have

$$\varepsilon_0 \frac{d^2\emptyset}{dx^2} = -en_\infty \exp\,(\frac{e\phi}{kT_e} - 1) \qquad (7)$$

In the region where $\left(\frac{e\phi}{kT_e}\right)$ <<1, we can expand the exponential in a Taylor series:

No simplification is possible for the region near the grid, where $|\frac{e\phi}{kT_e}|$ may be large. Fortunately, this region does not contribute much to the thickness of the cloud (called a sheath), because the potential falls very rapidly there. Keeping only the linear terms in Eq. (7), we have

$$\varepsilon_0 \frac{d^2\emptyset}{dx^2} = \frac{e^2 n_\infty}{kT_e}\phi \qquad (8)$$

Defining

$$\lambda_D = \left(\frac{\varepsilon_0 kT_e}{ne^2}\right)^{1/2} \qquad (9)$$

where n stands for n_∞, and KT_e is in joules. KT_e is often given in eV, in which case, we will write it also as T_{eV}.

We can write the solution of Eq. (1.14) as

$$\phi = \phi_0 \exp\,(-|x|/\lambda_D) \qquad (10)$$

The quantity λ_D, called the Debye length, is a measure of the shielding distance or thickness of the sheath.

1.4. Pinch Effect:

In view of the importance of plasma confinement by a magnetic field in controlled thermonuclear research, as well as in other applications, here we present a detailed treatment of plasma confinement for the special case in which the confinement is produced by an azimuthal (B) self-magnetic field, due to an axial current in the plasma generated by an appropriately applied electric field. Consider an infinite cylindrical column of conducting fluid with an axial current density J = $J_z(r)\, \hat{z}$ and a resulting azimuthal magnetic induction B = $B_\theta(r)\, \hat{\theta}$, as depicted in Fig. 1. The J × B force, acting on the plasma, forces the column to contract radially. This radial constriction of the plasma column is known as the pinch effect. In this case the isobaric surfaces, for which p = constant, are concentric cylinders. As the plasma is compressed radially, the plasma number density and the temperature increase. The plasma kinetic pressure counteracts to hinder the constriction of the plasma column, whereas the magnetic force acts to confine the plasma. When these counteracting forces are balanced, a steady-state condition results in which the plasma is mainly confined within a certain radius R, which remains constant in time. This situation is commonly referred to as the equilibrium pinch. When the self magnetic pressure exceeds the plasma kinetic pressure, the column radius changes with time, resulting in a situation known as the dynamic pinch. In what follows we investigate first the equilibrium pinch and afterwards the dynamic pinch.

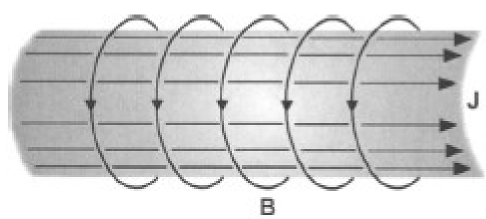

Fig. 1 Pinch configuration in which magneto plasma is confined by azimuthal magnetic fields generated by axial currents flowing along the plasma column.

1.5. Alfven Wave:

Alfve´n waves in plasma were first generated and detected by Allen, Baker, Pyle, and Wilcox at Berkeley, California, and by Jephcott in England in 1959.

An Alfven wave is a wave that occurs in a plasma or conducting fluid resulting from the interaction of the magnetic fields and electric currents within it, causing an oscillations of the ions.

If a conducting liquid is placed in a constant magnetic field, every motion of the liquid gives rise to an e.m.f. which produces electric currents. Owing to the magnetic field, these currents give mechanical forces which change the state of motion of the liquid. Thus a kind of combined electromagnetic hydrodynamic wave is produced. The work was done in hydrogen plasma created in a “slow pinch” discharge between two electrodes aligned along a magnetic field. Discharge of a slow capacitor bank A created the plasma.

Alfven waves initiated the field of magneto hydrodynamics which subsequently earned Alfven a Nobel prize.